Matrix model and string -Loop equation and continuum limit of IIB matrix model-

CFT and Integrability
In Memory Of Alexei Zamolodchikov

Dec 16 2013

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(Kyoto Univ.)

Alyosha's talks at YITP (Kyoto Univ.)

2001/10/24
Perturbed conformal field theory in spherical geometry
2005/06/03
Gravitational Sin-Gordon Model Revisited
2006/09/15
Metastability in 2D Liouville gravity
2007/08/27
Sinh-Gordon Boundary TBA and Boundary Liouville Reflection Amplitude

The poster of this conference:

"He regularly and thoroughly contributed to the development of Liouville string theory - the key model for the understanding of the most fundamental aspects of string theory and two-dimensional quantum gravity."

IIB matrix model

Dimensional reduction of 10D super Yang-Mills theory reduced to zero dimensions

$$S = -\frac{1}{g^2} \mathrm{Tr}\left(\frac{1}{4}[A^{\mu}, A^{\nu}]^2 + \frac{1}{2}\overline{\Psi}\Gamma^{\mu}[A^{\mu}, \Psi \ ] \right)$$

Discretization ↑

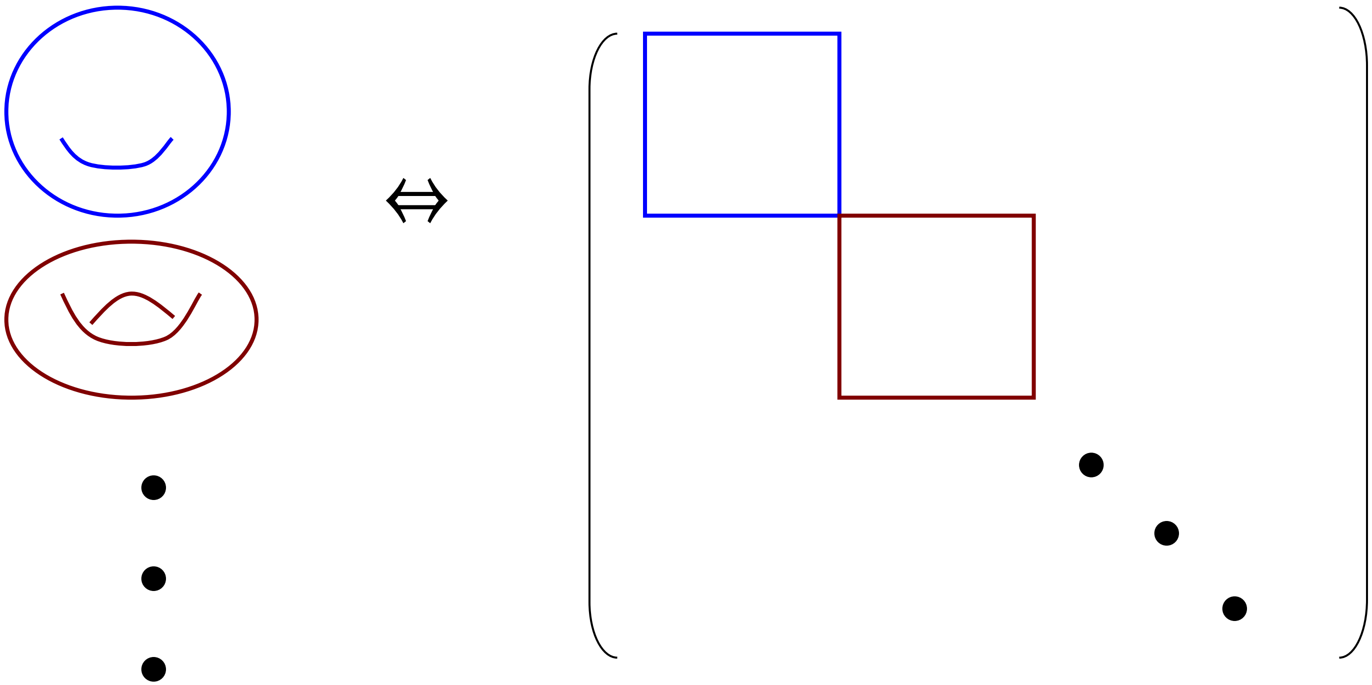
$$\{ \ , \ \} \rightarrow [\ , \ ]$$

$$\int \rightarrow Tr$$

Green-Schwartz action in the Schild Gauge

$$S = \int d^2\xi \left(\frac{1}{4}\{X^{\mu}, X^{\nu}\}^2 + \frac{1}{2}\overline{\Psi}\,\Gamma^{\mu}\{A^{\mu}, \Psi\} \right)$$

Expected to describe the infinite system of strings in the $N \to \infty$ limit.



Proof ?　We want to show that the string perturbation series is reproduced.

IR cutoff

Because of the supersymmetry, the force between the eigenvalues is cancelled:

$$S^{(1-loop)}{}_{eff} = (8-8)\sum_{i,j}\log\left(\left(p^{(i)}-p^{(j)}\right)^2\right)=0.$$

It *seems* that we have to impose an infrared cutoff to prevent the eigenvalues from running away to infinity.

$$-R/2 < \mathrm{eigen}\left(A^{\mu}\right) < R/2$$

Strictly speaking, the diagonal elements of the fermionic matrices are subtle.
The one-loop effective Lagrangian is given by

$$A_\mu = \begin{pmatrix} p_\mu^{(1)} & & * \\ & \ddots & \\ * & & p_\mu^{(N)} \end{pmatrix} \qquad \psi = \begin{pmatrix} \xi^{(1)} & & * \\ & \ddots & \\ * & & \xi^{(N)} \end{pmatrix}$$

$$S_{\mathrm{eff}}^{\text{1-loop}}(x,\xi) = \sum_{i<j} tr\left(\frac{S_{(i,j)}{}^4}{4} + \frac{S_{(i,j)}{}^8}{8} \right),$$

$$\left(S_{(i,j)}\right)_{\mu,\nu} = \left(\bar{\xi}^{(i)} - \bar{\xi}^{(j)}\right)\Gamma^{\mu\alpha\nu}\left(\xi^{(i)} - \xi^{(j)}\right)\frac{p_\alpha{}^{(i)} - p_\alpha{}^{(j)}}{\left(\left(p^{(i)} - p^{(j)}\right)^2\right)^2}.$$

The partition function becomes finite even if IR cutoff is not there.

But their effect is small compared to $O(N^2)$.

We simply impose IR cutoff in this discussion.

How to take large-N limit ?

A **are regarded as space-time coordinates:**

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A^\mu,\ A^\nu]^2 + \frac{1}{2}\overline{\Psi}\Gamma^\mu[A^\mu,\Psi\ \ ]\right).$$

$$\Rightarrow \quad \left[g^2\right] = L^4.$$

Question:

How are the Regge slope and string coupling expressed in terms of *g*, *N* and *R*?

No definite answer. One possibility is

$$\alpha' = \frac{g^2 N}{R^2}, \quad g_s = \frac{R^4}{\alpha'^2 N}.$$

Loop equation and light-cone string field

Light-cone string field recovers the string perturbation.

Does the matrix model reproduces the light-cone string field ?

rough expectation

Wilson loop = string field

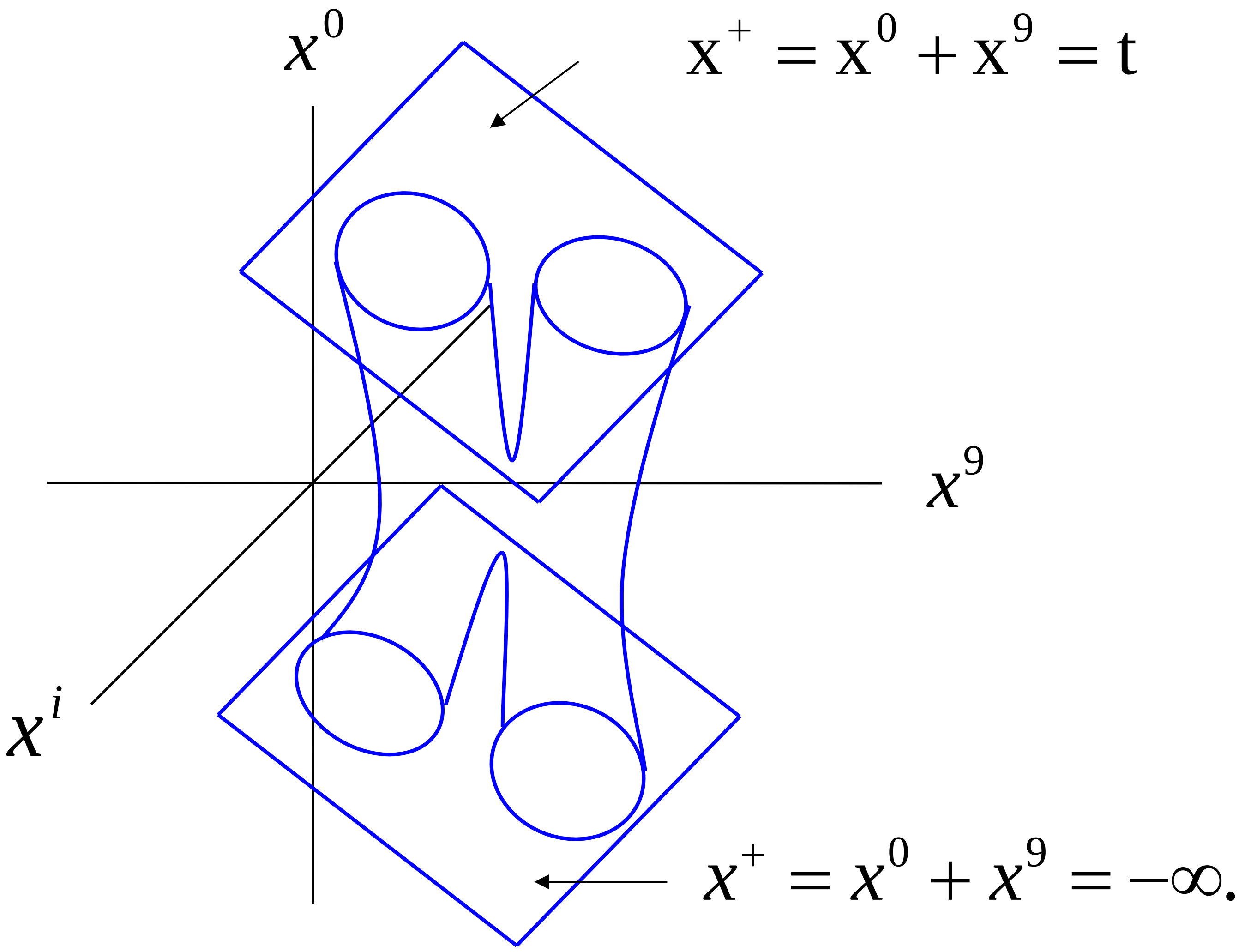
$$w\left[k_\mu\right] = Tr\left(P\exp\left(i\oint d\sigma\, k_\mu(\sigma) A^\mu + fermion\right)\right)$$

$$\updownarrow$$

Creation annihilation operator of $|\, k_\mu(\cdot\cdot) >$.

Conjecture **Wilson loop on the light front**

= light-cone string field



Loop equation

Only bosonic part, for simplicity. $S=-\frac{1}{4g^2}Tr\left[A^\mu,A^\nu\right]^2$

Wilson loop = string in momentum rep. $w[k]=Tr\left(P\exp\left(i\int_0^{2\pi}k_\mu(\sigma)A^\mu\right)\right)$

Schwinger –Dyson eq.

$$\frac{1}{g^2}\left(\frac{\delta}{\delta k^\mu(\sigma+0)}-\frac{\delta}{\delta k^\mu(\sigma-0)}\right)\left(\frac{\delta}{\delta k_\mu(\sigma)}\right)' < w[k]\,w[k^{(1)}]\cdots w[k^{(n)}] >$$

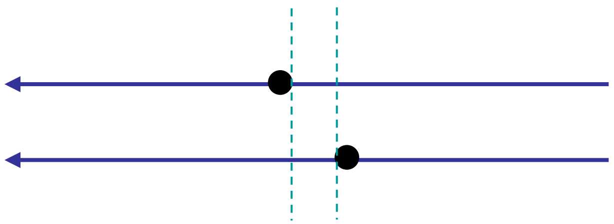
$$=\int_0^{2\pi}d\sigma'\,k^\mu(\sigma)\,k_\mu(\sigma') \;< w_{\sigma\sigma'}[k]\,w_{\sigma'\sigma}[k]\,w[k^{(1)}]\cdots w[k^{(n)}] >$$

$$+\sum_{i=1}^{n}\int_0^{2\pi}d\sigma'\,k^\mu(\sigma)\,k^{(i)}{}_\mu(\sigma') < Tr(w_\sigma[k]\,w_{\sigma'}[k^{(i)}])\,w[k^{(1)}]\cdots\underset{i}{\wedge}\cdots w[k^{(n)}] >$$

Meaning of LHS

$$\frac{\delta}{\delta k_\mu(\sigma)} P\exp\left(i\int_0^{2\pi} d\sigma'\, k(\sigma')\cdot A\right)$$

$$= i\, P\exp\left(i\int_\sigma^{2\pi} d\sigma'\, k(\sigma')\cdot A\right) A^\mu\; P\exp\left(i\int_0^{\sigma} d\sigma'\, k(\sigma')\cdot A\right)$$



$$\left(\frac{\delta}{\delta k_\mu(\sigma)}\right)' P\exp\left(i\int_0^{2\pi} d\sigma'\, k(\sigma')\cdot A\right)$$

$$= -\, P\exp\left(i\int_\sigma^{2\pi} d\sigma'\, k(\sigma')\cdot A\right)\left[A^\mu, k(\sigma)\cdot A\right] P\exp\left(i\int_0^{\sigma} d\sigma'\, k(\sigma')\cdot A\right)$$

$$\left(\frac{\delta}{\delta k_\mu(\sigma+0)} - \frac{\delta}{\delta k_\mu(\sigma-0)}\right)\left(\frac{\delta}{\delta k_\mu(\sigma)}\right)' P\exp\left(i\int_0^{2\pi} d\sigma'\, k(\sigma')\cdot A\right)$$

$$= -i\, P\exp\left(i\int_\sigma^{2\pi} d\sigma'\, k(\sigma')\cdot A\right)\left[A_\mu,\left[A^\mu, k(\sigma)\cdot A\right]\right] P\exp\left(i\int_0^{\sigma} d\sigma'\, k(\sigma')\cdot A\right)$$

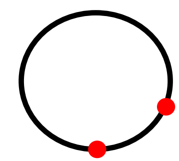
Schwinger-Dyson eq.

For $\delta^a_\mu A^\nu = t^a \delta^\nu_\mu$,

$$0 = \int dA\, \delta^a_\mu \left\{ Tr(t^a\ w_\sigma[k])\, w[k^{(1)}] \cdots w[k^{(n)}]\, e^{-S(A)} \right\}.$$

where $w_\sigma[k] = Pe^{i\int_0^\sigma d\sigma'\, k(\sigma')\cdot A} Pe^{i\int_\sigma^{2\pi} d\sigma'\, k(\sigma')\cdot A}$.

$$\delta^a_\mu\, Tr(t^a\ w_\sigma[k]) = i\int_0^{2\pi} d\sigma'\, k_\mu(\sigma')\ \ w_{\sigma\sigma'}[k]\, w_{\sigma'\sigma}[k]$$

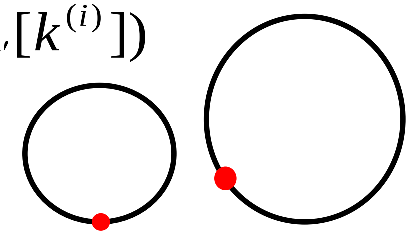


where

$$w_{\sigma_2\sigma_1}[k] = \begin{cases} Tr(Pe^{i\int_{\sigma_1}^{\sigma_2} d\sigma'\, k(\sigma')\cdot A}) & (\sigma_2 > \sigma_1) \\ Tr(Pe^{i\int_0^{\sigma_2} d\sigma'\, k(\sigma')\cdot A} Pe^{i\int_{\sigma_1}^{2\pi} d\sigma'\, k(\sigma')\cdot A}) & (\sigma_2 < \sigma_1) \end{cases}$$

$$Tr(t^a\ w_\sigma[k])\, \delta^a_\mu\ w[k^{(i)}] = i\int d\sigma'\, k^{(i)}{}_\mu(\sigma')\, Tr(w_\sigma[k]\, w_{\sigma'}[k^{(i)}])$$

$$Tr(t^a\ w_\sigma[k])\, \delta^a_\mu S = \frac{1}{g^2} Tr([A_\nu,[A^\nu, A_\mu]]\, w_\sigma[k])$$



Free string

We want to consider freely propagating string.

$$\frac{1}{g^2}\left(\frac{\delta}{\delta k^\mu(\sigma+0)}-\frac{\delta}{\delta k^\mu(\sigma-0)}\right)\left(\frac{\delta}{\delta k_\mu(\sigma)}\right)' < w[k]\, \underbrace{w[k^{(1)}]\cdots w[k^{(n)}]} >$$

Regard as spectators.

$$= \int_0^{2\pi} d\sigma'\, k^\mu(\sigma)\, k_\mu(\sigma') \;< w_{\sigma\sigma'}[k]\, w_{\sigma'\sigma}[k]\, w[k^{(1)}]\cdots w[k^{(n)}] >$$

splitting

$$+\sum_{i=1}^{n}\int_0^{2\pi} d\sigma'\, k^\mu(\sigma)\, k^{(i)}{}_\mu(\sigma') < Tr(w_\sigma[k]\, w_{\sigma'}[k^{(i)}])\, w[k^{(1)}]\cdots \underset{i}{\wedge} \cdots w[k^{(n)}] >$$

merging

Naively, we have

$$\left(\frac{\delta}{\delta k^\mu(\sigma+0)}-\frac{\delta}{\delta k^\mu(\sigma-0)}\right)\left(\frac{\delta}{\delta k_\mu(\sigma)}\right)' < w[k]\, w[k^{(1)}]\cdots w[k^{(n)}] > = 0$$

Problem 1 $\quad (k(\sigma))^2 < w[k]\, w[k^{(1)}]\cdots w[k^{(n)}] >$ is missing.

Problem 2 $\quad \left(\frac{\delta}{\delta k^\mu(\sigma+0)}-\frac{\delta}{\delta k^\mu(\sigma-0)}\right)\left(\frac{\delta}{\delta k_\mu(\sigma)}\right)' \sim 0\cdot(X'(\sigma))^2$

Possible solution for the problem 1:

If we have $< w_{\sigma\sigma'}[k] > = c\delta(\sigma - \sigma') + \cdots,$

$$\int_0^{2\pi} d\sigma' \, k^{\mu}(\sigma) \, k_{\mu}(\sigma') \; < w_{\sigma\sigma'}[k] \, w_{\sigma'\sigma}[k] \, w[k^{(1)}] \cdots w[k^{(n)}] >$$

$$= c\left(k(\sigma)\right)^2 < w[k] \, w[k^{(1)}] \cdots w[k^{(n)}] > + \cdots$$

Possible solution for the problem 2:

If an effective cutoff ε appears in the σ space, we have

$$\left(\frac{\delta}{\delta k^{\mu}(\sigma+0)} - \frac{\delta}{\delta k^{\mu}(\sigma-0)}\right)\left(\frac{\delta}{\delta k_{\mu}(\sigma)}\right)' \sim -\varepsilon \cdot \left(X'(\sigma)\right)^2 .$$

Natural guess from the transformation of scalar density:

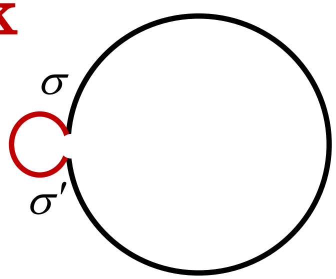
$$c = \frac{c_0(N,R)}{\sqrt{k(\sigma)^2}} \qquad\qquad \varepsilon = \frac{\varepsilon_0(N,R)}{\sqrt{k(\sigma)^2}}$$

naive estimation of c

$$<w_{\sigma\sigma'}[k]> = < Tr(e^{i\int_{\sigma}^{\sigma'} d\sigma'' k(\sigma'')\cdot A}) >$$ ← **small disk**

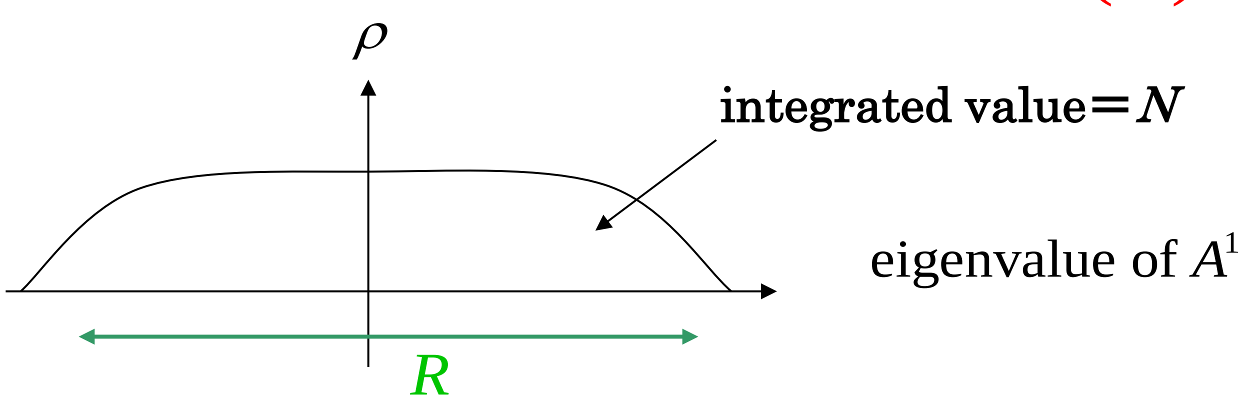
$$\sim < Tr(e^{i(\sigma'-\sigma)k(\sigma)\cdot A}) > \sim < Tr(e^{i(\sigma'-\sigma)\sqrt{k(\sigma)^2}\cdot A^1}) >$$

$$= \int d\lambda\, \rho(\lambda)\, e^{i(\sigma'-\sigma)\sqrt{k(\sigma)^2}\lambda} \sim c\delta(\sigma'-\sigma)$$



$$c = \frac{2\pi\rho(0)}{\sqrt{k(\sigma)^2}} \sim \frac{2\pi N}{R\sqrt{k(\sigma)^2}}$$

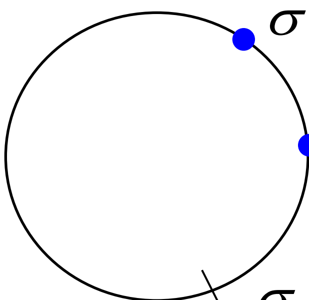
What if $k(\sigma)^2 = 0$?

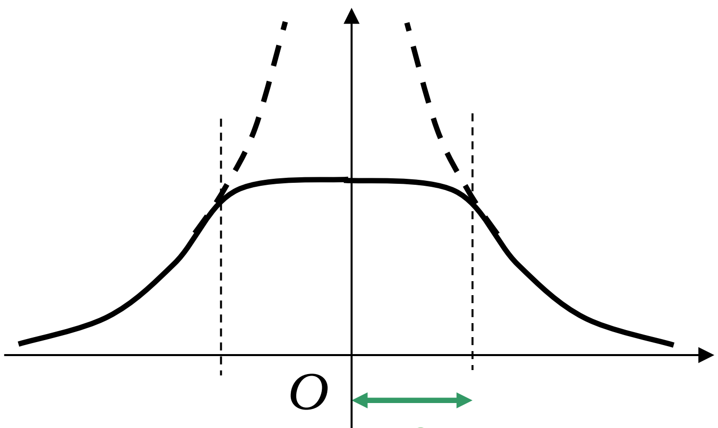


naive guess for ε

$$G(\sigma_1,\sigma_2)$$

$$=< Tr\left(Pe^{i\int_{\sigma_2}^{2\pi} d\sigma' k(\sigma')\cdot A} A^\mu Pe^{i\int_{\sigma_1}^{\sigma_2} d\sigma' k(\sigma')\cdot A} A^\nu Pe^{i\int_0^{\sigma_1} d\sigma' k(\sigma')\cdot A} \right) >$$





$$e^{i\int_{\sigma_1}^{\sigma_2} d\sigma k(\sigma)\cdot A} \sim e^{i(\sigma_2-\sigma_1)k(\sigma_1)\cdot A} \sim 1 \quad \Leftrightarrow \quad |\sigma_2 - \sigma_1|\sqrt{k(\sigma_1)^2} R \le 1$$

$$\Rightarrow \quad \varepsilon > \frac{1}{R\sqrt{k(\sigma)^2}} \qquad \varepsilon \sim \frac{1}{R\sqrt{k(\sigma)^2}} \ ?$$

Then the loop equations become

$$-\frac{1}{\sqrt{k(\sigma)^2}}\left(\frac{\varepsilon_0}{g^2}\left(X'(\sigma)\right)^2 + c_0\left(k(\sigma)\right)^2\right) < w[k]\, w[k^{(1)}]\cdots w[k^{(n)}] >$$

free propagation

$$= \int_0^{2\pi} d\sigma'\, k^{\mu}(\sigma)\, k_{\mu}(\sigma') \;< w_{\sigma\sigma'}[k]\, w_{\sigma'\sigma}[k]\, w[k^{(1)}]\cdots w[k^{(n)}] > .$$

splitting

$$+\sum_{i=1}^{n}\int_0^{2\pi} d\sigma'\, k^{\mu}(\sigma)\, k^{(i)}{}_{\mu}(\sigma') < Tr(w_{\sigma}[k]\, w_{\sigma'}[k^{(i)}])\, w[k^{(1)}]\cdots \underset{i}{\wedge} \cdots w[k^{(n)}] >$$

merging

The free part

$$\left(\frac{\varepsilon_0}{g^2}\left(X'(\sigma)\right)^2 + c_0\left(k(\sigma)\right)^2\right) < w[k]\, w[k^{(1)}]\cdots w[k^{(n)}] >= 0$$

$$\Leftrightarrow L_n < w[k]\, w[k^{(1)}]\cdots w[k^{(n)}] >= 0 \quad (\text{for } \forall n)$$

does not give a normalizble functional.

The operator insertion at the splitting or merging point should be clarified.

Wilson loops on light front

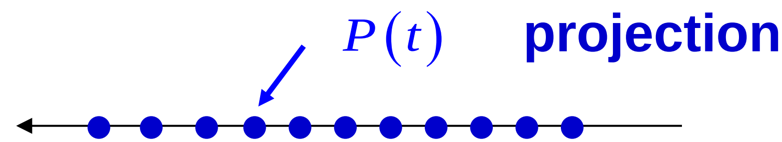
$$w[k^-,k^+,k^i]=Tr(Pe^{i\int_0^{2\pi}d\sigma\,(-k^+(\sigma)A^- - k^-(\sigma)A^+ + k^i(\sigma)A^i)})$$

$$\tilde{w}[x^+,k^+,k^i]=\int[dk^-]\,e^{i\int_0^{2\pi}d\sigma\,k^-(\sigma)x^+(\sigma)}w[k^-,k^+,k^i]$$

(1) $k^+(\sigma)>0 \Rightarrow k^+(\sigma)=1,\ \int_0^{2\pi}d\sigma \to \int_0^{p^+}d\sigma\ ,\ p^+=\int_0^{2\pi}d\sigma\,k^+(\sigma)$.

rep. inv.

(2) $x^+(\sigma)=t$ (const) $\leftarrow$ **Wilson loops on light front**



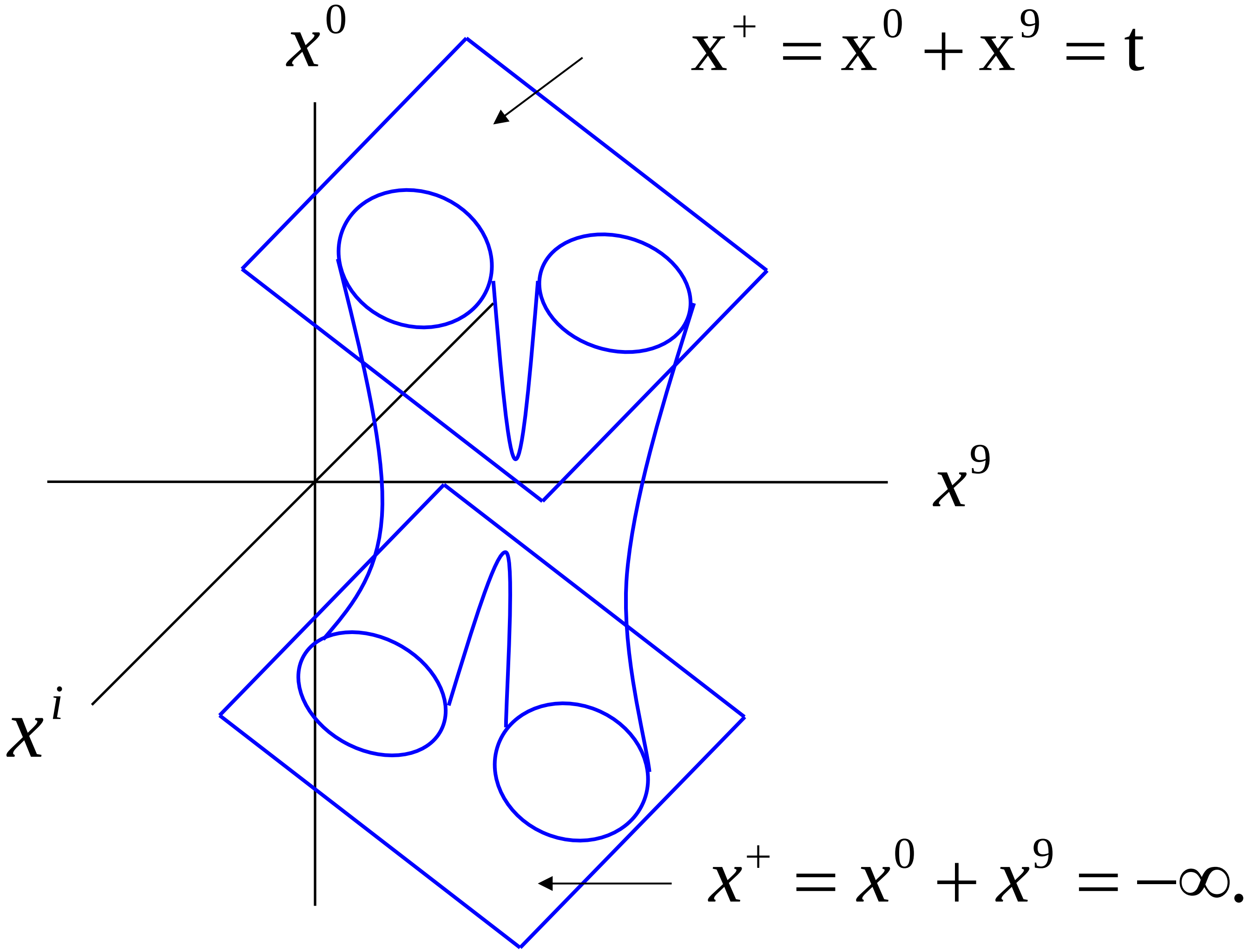
$$w(t,p^+,k^i(\,))$$

$$=\int[dk^-]\,e^{i\int_0^{p^+}d\sigma\,k^-(\sigma)t}\,Tr(Pe^{i\int_0^{p^+}d\sigma\,(-A^- - k^-(\sigma)A^+ + k^i(\sigma)A^i)})$$

$$=Tr(PP(t)e^{i\int_0^{p^+}d\sigma\,(-P(t)A^-P(t)+k^i(\sigma)P(t)A^iP(t))})$$

$$P(t)=\delta(0)^{-1}\delta(A^+-t)\ \Rightarrow\ \theta\left(\left|A^+-t\right|<\eta\right)$$

Put the Wilson loops on a light front:



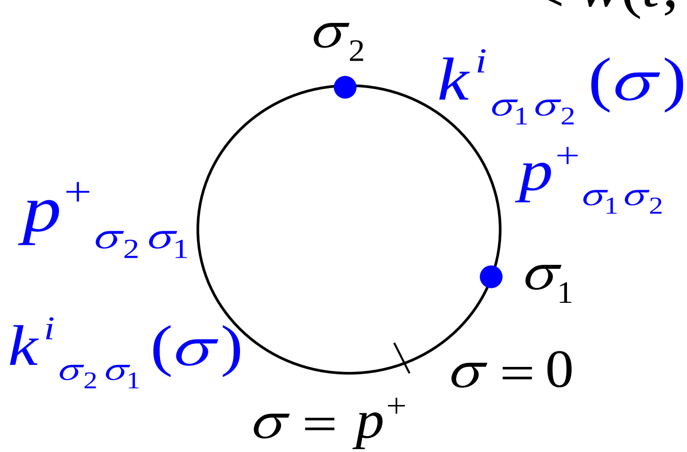
loop equations in this set up

$$\text{LHS} = \frac{1}{g^2}\left(\frac{\delta}{\delta k^i(\sigma+0)} - \frac{\delta}{\delta k^i(\sigma-0)}\right)\left(\frac{\delta}{\delta k^i(\sigma)}\right)' < w(t, p^+, k^i(\,))\cdots >$$

RHS

$$\text{1st term} = \int_0^{p^+} d\sigma' \left(i\frac{\delta}{\delta x^+(\sigma)} + i\frac{\delta}{\delta x^+(\sigma')} + k^i(\sigma)k^i(\sigma') \right)$$

$$\cdot < w(t,\ p^+_{\sigma\sigma'}, k^i_{\sigma\sigma'}(\,))\ w(t,\ p^+_{\sigma'\sigma}, k^i_{\sigma'\sigma}(\,))\cdots >$$



$$p^+_{\sigma_1\sigma_2} = \begin{cases} \sigma_2 - \sigma_1 & \text{for } \sigma_2 > \sigma_1. \\ p^+ + \sigma_2 - \sigma_1 & \text{for } \sigma_2 < \sigma_1. \end{cases}$$

$$k^i_{\sigma_1\sigma_2}(\sigma) = k^i(\sigma + \sigma_1),\ \ 0 < \sigma < p^+_{\sigma_1\sigma_2}.$$

$$k^i(\sigma + p^+) = k^i(\sigma).$$

2nd term **Similarly.**

small disk

$$< w(t,\, p^{+}{}_{\sigma\sigma'},\, k^{i}{}_{\sigma\sigma'}(\,))\; w(t,\, p^{+}{}_{\sigma'\sigma},\, k^{i}{}_{\sigma'\sigma}(\,)) \cdots >$$

$$=< w(t,\, p^{+}{}_{\sigma\sigma'},\, k^{i}{}_{\sigma\sigma'}(\,)) >< w(t,\, p^{+}{}_{\sigma'\sigma},\, k^{i}{}_{\sigma'\sigma}(\,)) \cdots > + \cdots$$

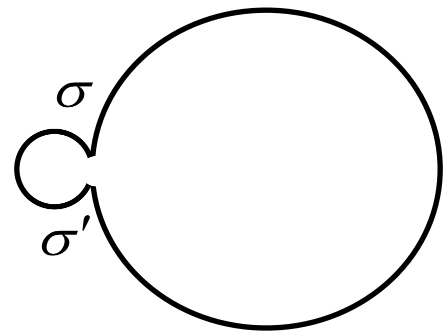
$$< w(t,\, p^{+}{}_{\sigma\sigma'},\, k^{i}{}_{\sigma\sigma'}(\,) = 0) >$$

$$=< Tr\left(PP(t)\, e^{-i(\sigma'-\sigma)P(t)A^{-}P(t)} \right) >$$

$$= \int d\lambda\, \bar{\rho}(\lambda)\, e^{i(\sigma'-\sigma)\lambda}$$

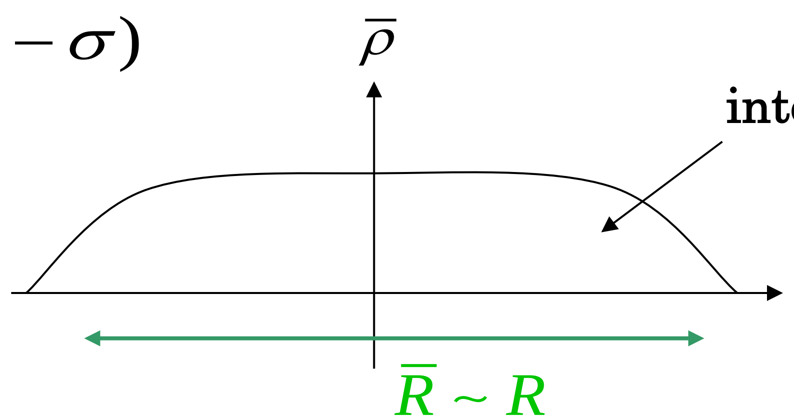
$$\simeq 2\pi\bar{\rho}(0)\delta(\sigma'-\sigma)$$

$$\simeq \underbrace{\frac{2\pi N\eta}{R^2}}_{I}\delta(\sigma'-\sigma)$$



$(\sigma' > \sigma)$

$$\bar{N} = Tr\theta\left(\left|A^{+} - t\right| < \eta\right) = N\frac{\eta}{R}$$



kinetic term generated by the small disk

$$\int d\sigma' F(\sigma') < w(t,\ p^{+}{}_{\sigma\sigma'},\ k^{i}{}_{\sigma\sigma'}(\ ))\ w(t,\ p^{+}{}_{\sigma'\sigma},\ k^{i}{}_{\sigma'\sigma}(\ )) \cdots > + \cdots$$

$$= I\, F(\sigma) < w(t,\ p^{+},\ k^{i}(\ )) \cdots > + \cdots$$

RHS 1st term $= I \left(2i \frac{\delta}{\delta x^{+}(\sigma)} + \left(k^{i}(\sigma)\right)^{2} \right) < w(t,\ p^{+},\ k^{i}(\ )) \cdots > + \cdots$

Assuming the existence of an "effective cutoff",

LHS $= \frac{1}{g^{2}} \left(\frac{\delta}{\delta k^{i}(\sigma+0)} - \frac{\delta}{\delta k^{i}(\sigma-0)} \right) \left(\frac{\delta}{\delta k^{i}(\sigma)} \right)' < w(t, p^{+}, k^{i}(\ )) \cdots >$

$$= \frac{\varepsilon}{g^{2}} \left(\left(\frac{\delta}{\delta k^{i}(\sigma)} \right)' \right)^{2} < w(t, p^{+}, k^{i}(\ )) \cdots > = -\frac{\varepsilon}{g^{2}} \left(X_{i}'(\sigma) \right)^{2} < w(t, p^{+}, k^{i}(\ )) \cdots > .$$

The loop equations become

$$-i\frac{\delta}{\delta x^{+}(\sigma)} < w(t,\, p^{+}, k^{i}(\,))\cdots >$$

$$= \left(\frac{1}{2}\left(k^{i}(\sigma)\right)^{2} + \frac{1}{2}\underbrace{\frac{\varepsilon}{g^{2}I}}_{\alpha'^{-2}} X_i'(\sigma)^{2}\right) < w(t,\, p^{+}, k^{i}(\,))\cdots >$$

free part

$$+\underbrace{\frac{1}{2I\varepsilon}}_{g_{st}\,\alpha'}\int_0^{p^{+}} d\sigma' \varepsilon\, k_i(\sigma)\, k_i(\sigma') < w(t,\, p^{+}_{\sigma\sigma'}, k^{i}_{\sigma\sigma'}(\,))w(t,\, p^{+}_{\sigma'\sigma}, k^{i}_{\sigma'\sigma}(\,))\cdots > + \cdots$$

interaction part

This reproduces the light-cone Hamiltonian after integrated over σ.

α' : Regge slope

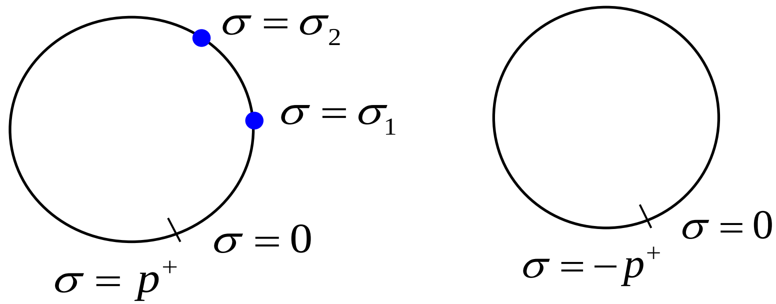
g_{st} : string coupling

$$\alpha'^{2} \sim \frac{g^{2}I}{\varepsilon} \qquad I \sim \frac{N\eta}{R^{2}}$$

$$g_{st}\alpha' \sim \frac{1}{I\varepsilon}$$

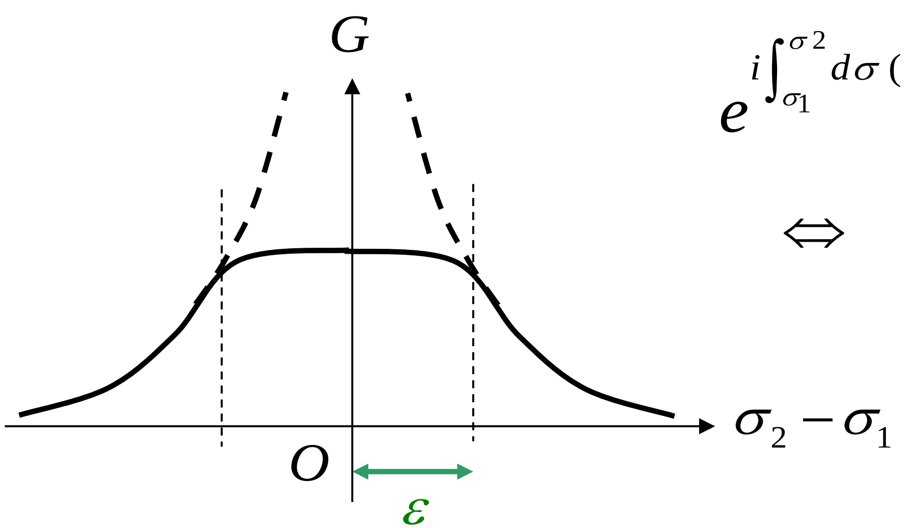
effective cutoff for σ axis δ

$$G(\sigma_1,\sigma_2) = < w(t, p^+, 0\ ; \sigma_1, O_1\ ; \sigma_2, O_2)\ \ w(t', -p^+, 0) >$$



$$w(t, p^+, 0\ ; \sigma_1, O_1\ ; \sigma_2, O_2)$$

$$= Tr(PP(t) e^{i\int_{\sigma_2}^{p^+} d\sigma\,(-P(t)A^- P(t))} O_2 e^{i\int_{\sigma_1}^{\sigma_2} d\sigma\,(-P(t)A^- P(t))} O_1 e^{i\int_{0}^{\sigma_1} d\sigma\,(-P(t)A^- P(t))})$$



$$e^{i\int_{\sigma_1}^{\sigma_2} d\sigma\,(-P(t)A^- P(t))} \sim e^{-i(\sigma_2-\sigma_1)P(t)A^- P(t)} \sim 1$$

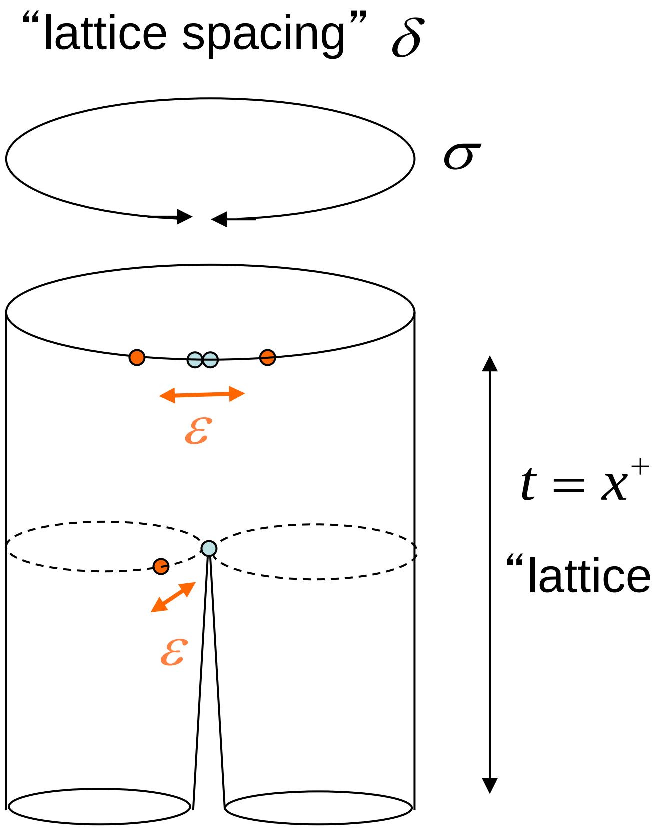
$$\Leftrightarrow \quad |\sigma_2 - \sigma_1| R \le 1$$

$$\Rightarrow \quad \delta > \frac{1}{R} \qquad \delta \sim \frac{1}{R}?$$

Relation between ε and η.

δ : "lattice spacing" for σ

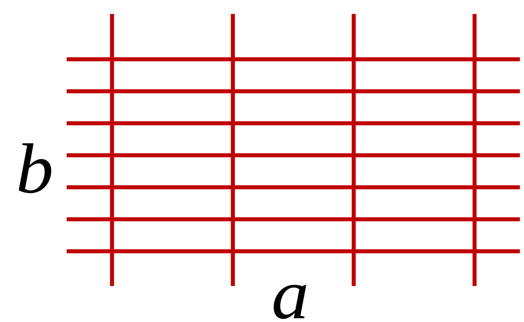
η is a "lattice spacing" for t .



The effective cutoff on the world sheet is given by the larger one:

$$\varepsilon = \max\left(\delta, \frac{\eta}{\alpha'}\right).$$

If $\alpha'\delta < \eta$, $\quad \varepsilon \sim \dfrac{\eta}{\alpha'}$.



Regge slope and string coupling

$$\alpha'^2 \sim g^2 \frac{I}{\varepsilon} \sim g^2 \frac{N\eta}{R^2}\frac{\alpha'}{\eta} \sim g^2 N \frac{\alpha'}{R^2} \Rightarrow \alpha' = \frac{g^2 N}{R^2}$$

$$g_{st} \sim \frac{1}{I\varepsilon\alpha'} \sim \left(\frac{N\eta}{R^2}\frac{\eta}{\alpha'}\alpha'\right)^{-1} \sim \frac{1}{N}\frac{R^2}{\eta^2}$$

Parameters of the action: $g,\ N.$

Parameter of the vacuum: $R.$

Parameter of the definition of light-cone Wilson loop: $\eta.$

$(\varepsilon \sim \frac{\eta}{\alpha'})$

$\Leftrightarrow$

Regge slope: $\alpha'.$

String coupling: $g_{st}.$

Speculation on $\eta \to 0$ limit

The cutoff of in the time direction smaller than $\alpha'\delta$,
does not make sense.

$\Rightarrow$ The $\eta \to 0$ limit is saturated by $\eta = \alpha'\delta$.

$$g_{st} \sim \frac{1}{N}\frac{R^2}{\eta^2} \to \frac{1}{N}\left(\frac{R}{\alpha'\delta}\right)^2$$

If $\delta \sim \frac{1}{R}$, then $g_{st} \sim \frac{1}{N}\left(\frac{R^2}{\alpha'}\right)^2$.

Summary

Matrix integral of IIB matrix model may be calculable by using the loop equations.

The double scaling limit can be taken such as

$$\alpha' = \frac{g^2 N}{R^2}, \quad g_s = \frac{R^4}{\alpha'^2 N}.$$

However, we need better understanding about the effective cutoff.

Relation to integrable systems?

For noncritical strings,
Loop equations ⇔ Virasoro constraint
⇔ KP hierarchy.