On the dynamic incentive of price-quality differentiation by a monopolist firm

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Abstract

When consumers are heterogeneous in their preferences about the quality of a product, a monopolist firm can take advantage of this heterogeneity, thereby, increase the profit by offering different price-quality pairs. This business practice is called the second price discrimination or non-linear pricing. This paper extends the static non-linear pricing problem into the dynamic one where the monopolist firm cannot precommit in advance. The main result is that the dynamic non-linear pricing outcome is the same as the static non-linear pricing outcome so that additional opportunities to transact neither benefits nor hurts the monopolist firm.

1. Introduction

When consumers are heterogeneous in that their preferences about the quality of a product differ, a monopolist firm can take advantage of this heterogeneity, thereby, increase the profit by offering many price-quality pairs called a menu instead of offering
a single quality. For example, take an automobile. Basically, every automobile functions the same as a transportation means. But, different consumers have different
valuations about the quality of automobile. Some consumers care about the basic function of the automobile only so that they want to buy the basic model with cheaper
price. Other consumers, in addition to the basic function, cares also about how the

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automobile look like so that they are willing to pay higher price for fancy cars like sports car. Actually, in real life, we observe that lots of different brands of automobiles are provided by the same automobile producer. Even within the same brand name, the quality differs, depending upon whether it has a power steering, windows, air bag etc.

This business practice is called in Economics the second degree price discrimination or non-linear pricing. In the literature, the pioneering work was that of Mussa and Rosen (1978). They examined the basic model where the monopolist firm produces a single product and the consumer's characteristic is one-dimensional. Since their work, the model was extended to the models with either multiproduct or multiple characteristics or both.[1)]

Although the models differ in details, these models share the common feature that the analysis is based upon the assumption that the monopolist can offer on a take-it-or-leave-it basis. In case of non-durable goods, this assumption seems a reasonable approximation of the reality. In the case of durable goods such as automobiles, however, it is dubious whether the monopolist firm has indeed such a strong commitment power. This paper addresses the issue of what would happen if the monopolist cannot commit. What will be the equilibrium if the monopolist can offer another menu in the next period once the current menu is rejected? In other words, this paper extends the static non linear pricing problem to the dynamic one. In case of the durable goods, in particular, these seem to be economically important and relavant quetions.

In static analysis, the consumers simply face the trade-off between price and quality, i.e., higher quality with higher price or lower quality with cheaper price. In dynamic analysis, one more factor comes in, the purchasing time. The consumer prefers higher quality, lower price and earlier purchasing time due to discounting. The dynamic

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1) For example, for the models with multiproduct and one-dimensional characteristic, see Roberts (1979), Mirman and Sibley (1980) and Mathews and Moore (1987). For the models with single product and multidimensional characteristic, see Laffont, Maskin, and Rochet (1987), and Lewis and Sappington (1988). For models with both multiproduct and multidimensional characteristic, see Spence (1980), McAfee and McMillan (1988), Armstrong (1986) and Rochet, Chone (1998). The book by Wilson (1993) is entirely devoted to the issue of non linear pricing.

non
linear pricing may potentially become more complicated because the monopolist firm should take into consideration the trade-off among three factors instead of two.

For the dynamic analysis, we assume the following extensive form game; In every period, the monopolist firm offers a menu consisting of several price-quality pairs. If the consumer accepts the menu and chooses a price-quality pair in it, the game ends. If the consumer rejects the menu, the game moves next period and the monopolist firm can offer another menu. The game continues until the consumer chooses a price-quality pair. The main result is that the equilibrium without commitment is exactly the same as one with commitment. In other words, the static non-linear pricing equilibrium outcome remains the unique equilibrium outcome in dynamic game as well. A consumer with high valuation is not only willing to pay higher price for the same increase in quality but also is more impatient, namely, wants to buy earlier than a consumer with lower valuation. The monopolist firm can separate the heterogeneous consumers either using the differences in willingness to pay or using impatience. The result shows that although the second method is now available in dynamic model, that does not help to increase the monopolist's profit. The result also justifies the static analysis based upon the unrealistic assumption of commitment power.

The paper is organized as follows. Section 2 presents a basic model. Section 3 briefly analyzes the static price-quality differentiation game. In Section 4, as an intermediate step, finite horizon price-quality differentiation game is analyzed and in Section 5, based upon the result in finite horizon game, the infinite horizon game is analyzed. Then, conclusion follows.

2. Model

A monopolist sells an indivisible product to a heterogeneous population of consumers. Each consumer wants to buy at most one unit of the product. The product can be of various qualities. The quality may in principle have multidimensional attributes. For

the
simplicity of analysis, however, we assume that the quality is one dimensional and is
denoted by q. q belongs to an interval, $[0, \infty)$. The lowest quality level is normalized to 0. The production technology is assumed to exhibit constant returns to scale with
respect to quality; the cost of producing a product with quality q is given by
$C(q) = cq$.

Depending upon his type, a consumer has a different valuation about the product with quality q. Concretely, we assume that θ-type consumer has an utility function
$\theta U(q)$ when he consumes the product with quality q. We assume that the whole population consists of two types of consumer. Namely, θ takes two values, θ_1 and
θ_2
with $0 < \theta_2 < \theta_1$. Call θ_1-type consumer a high value consumer, and θ_2-type consumer a low value consumer, respectively. Let μ_θ and $1-\mu_\theta$ be the proportions of high value consumers and low valuations consumers in the population, respectively. θ is private information to the consumer. Each consumer knows his type, but the monopolist cannot identify which consumer is which. It only knows the distribution.[2] On the shape of $U(\cdot)$, we make the following standard assumptions.

Assumption. $U(\cdot)$ is twice continuously differentiable with $U'>0$
and $U''<0$. $U(0)=0, U'(0)=\infty$ and $U'(\infty)=0$.

In other words, each consumer prefers higher q and shows diminishing marginal utility with respect to q.

For the purpose of later analysis, the first best level of q is defined as that which maximizes the surplus under complete information about θ. Consider the following
maximization problem:

$$\text{Max}_q \quad S(q;\theta) = \theta U(q) - cq .$$

By Assumption, $q^*(\theta) = \arg\max S(q;\theta)$ exists and uniquely determined by the first order condition; $\theta U'(q^*(\theta)) = c$.

We now specify the way the transaction occurs between the monopolist and the

2) Equivalently, there is one consumer whose type is of high value with probability μ_θ and is of low value with probability $1-\mu_\theta$. The consumer knows his type, but the monopolist knows the distribution only. We use the heterogeneous population interpretation and incomplete information interpretation interchangeably.

consumer. In each period, the monopolist offers a menu consisting of multiple price-quality pair. A price-quality pair (p,q) specifies both the price that the consumer pays and the quality of the product. The consumer can choose a price-quality pair from the current menu. If he chooses a particular price-quality pair, the game ends. If he rejects the menu, then in the next period the monopolist again offers a menu, and so on. The game continues indefinitely until the consumer chooses a price-quality pair. If the consumer chooses a price-quality pair (p,q) in period t, we denote this outcome by (p,q,t). When this happen, $\delta^{t-1}(p-cq)$ and $\delta^{t-1}(\theta U(q)-p)$ are the payoffs to the monopolist and the θ-type consumer, respectively. If the consumer rejects every menu offered, zero payoffs are assigned to both parties.

Since the population consists of two types of consumers, without loss of generality, we assume that in each period the monopolist offers a menu of two price-quality pairs. A history of length t, h_t consists of rejected menus up to period t; $h_t=(M_1,M_2,\cdots,M_t)$, where $M_t=\{(p_1^t,q_1^t),(p_2^t,q_2^t)\}$. Let H_t be the set of all histories of length t. As convention, we set $H_0=\{\emptyset\}$.

A pure strategy of the monopolist is denoted by σ^M, where $\sigma^M(h_{t-1})=\{(p_1^t,q_1^t),(p_2^t,q_2^t)\}$. Namely, conditional on h_{t-1}, in period t the monopolist offers a menu, $\{(p_1^t,q_1^t),(p_2^t,q_2^t)\}$. A pure strategy of the consumer is denoted by σ^C, where $\sigma^C(\{(p_1^t,q_1^t),(p_2^t,q_2^t)\} \mid h_{t-1},\theta)\in\{No,(p_1^t,q_1^t),(p_2^t,q_2^t)\}$. Namely, in period t, once he is offered a menu, $\{(p_1^t,q_1^t),(p_2^t,q_2^t)\}$, depending upon the previous history h_{t-1} and his type θ, the consumer either rejects the menu denoted by 'No' or accepts it and chooses one pair in it.

A system of beliefs is denoted by μ. $\mu(h_t)$ indicates, conditional on h_t, the proportion of high value consumers in the whole population or the posterior belief of the monopolist that the consumer is of high value, . A sequential equilibrium consists of a strategy profile and a system of beliefs which satisfy sequential rationality and consistency. Sequential rationality and consistency are defined in the usual manner. In the sequel, a sequential equilibrium is referred to simply as an equilibrium.

3. Static Price-Quality Differentiation Game

In this section, we consider the static price-quality differentiation problem which is known as a static non-linear pricing. The monopolist can offer a menu on a take-it-or-leave-it basis. If the consumer rejects the menu, the game ends and

both parties get zero payoff. If the consumer chooses (p, q) , $p-cq$ and $\theta U(q)-p$ are the
payoffs to the monopolist and the θ -type consumers, respectively.

Since static non-linear pricing problem is by now a well-known problem in the literature, we analyze it in a brief way. Let (p_i, q_i) be a price-quality pair intended for θ_i -type consumer, $i=1,2$. Then, the monopolist faces both participation and incentive constraint. For the participation constraints, $\theta_i U(q)-p_i \geq 0,$ $i=1,2$. For the incentive constraints, $\theta_i U(q_i)-p_i \geq \theta_i U(q_j)-p_j, i \neq j.$
It is well-known that among four constraints, only the participation constraint for the low value consumer and the incentive constraint for the high value consumer are binding at the optimum;
i.e., $p_2=\theta_2 U(q_2)$ and $\theta_1 U(q_1)-p_1=\theta_1 U(q_2)-p_2,$ therefore, $p_1=\theta_1 U(q_1)-(\theta_1-\theta_2)U(q_2).$
Hence the objective function of the monopolist is as follows:

$$\begin{aligned}\Pi &= \mu_0(p_1-cq_1)+(1-\mu_0)(p_2-cq_2) \\ &= \mu_0\{\theta_1 U(q_1)-(\theta_1-\theta_2)\theta_2 U(q_2)-cq_1\}+(1-\mu_0)\{\theta_2 U(q_2)-cq_2\}\end{aligned}$$.

Concavity of $U(\cdot)$ dictates that Π is also concave so that the first order conditions completely characterize the optimum.

$$\frac{\partial \Pi}{\partial q_1}=0 \Rightarrow \theta_1 U'(q_1)=c \qquad ---(1)$$

$$\frac{\partial \Pi}{\partial q_2}=0 \Rightarrow \theta_2 U'(q_2)=\frac{c}{1-\frac{\mu_0}{1-\mu_0}\frac{\theta_1-\theta_2}{\theta_2}} \qquad ---(2)$$

By Equation (1), q_1 equals the first best level, $q^*(\theta_1)$. Since $1-\frac{\mu_0}{1-\mu_0}\frac{\theta_1-\theta_2}{\theta_2}<1$, q_2 satisfying Equation (2) should be strictly less than $q^*(\theta_2)$. q_2 satisfying Equation
(2) depends upon μ_0 . In order to emphasize this dependence, we write down q_2 satisfying Equation (2) as $q_2(\mu_0)$. Note that $1-\frac{\mu_0}{1-\mu_0}\frac{\theta_1-\theta_2}{\theta_2}$ can be negative.

Let μ^* be the value of μ_0 which makes $1-\frac{\mu_0}{1-\mu_0}\frac{\theta_1-\theta_2}{\theta_2}=0$, i.e., $\mu^*=\frac{\theta_2}{\theta_1}$.

If $\mu_0<\mu^*$, $q_2(\mu_0)$ is defined by Equation (2). If $\mu_0 \geq \mu^*$, however, there should arise
the corner solution. For all $\mu_0 \geq \mu^*$, $q_2(\mu_0)=0.$

In the static price-quality differentiation game, the monopolist offers a menu, $\{(\theta_1 U(q^*(\theta_1))-(\theta_1-\theta_2)U(q_2(\mu_0)), q^*(\theta_1)), (\theta_2 U(q_2(\mu_0)), q_2(\mu_0))\}$, and θ_1 -type (θ_2 -type) consumer chooses the first (second) price-quality pair, respectively. The menu consisting of these two price-quality pairs is called the *static equilibrium menu*. The payoff to the monopolist from the static equilibrium menu is called the *static equilibrium payoff* and denoted by $\Pi_s(\mu_0)$. These results are summarized in Proposition 1.

Proposition 1. In static price-quality differentiation game, the monopolist offers the menu, $\{(\theta_1 U(q^*(\theta_1))-(\theta_1-\theta_2)U(q_2(\mu_0)), q^*(\theta_1)), (\theta_2 U(q_2(\mu_0)), q_2(\mu_0))\}$.

4. Dynamic Price-Quality Differentiation Game

In the previous section, the monopolist can make a take-it-or-leave-it offer. In this section, we assume away this commitment power. Once the menu is rejected, the game moves to the next period and the monopolist can offer another menu.

We consider two cases separately. The first case is the finite horizon price-quality differentiation game where there is a fixed number of periods, $T<\infty$ so that the game should end by period T . If the monopolist and the consumer do not reach an agreement by period T , both parties are assigned zero payoff. The second one is the infinite horizon price-quality differentiation game where the game continues until the consumer chooses a price-quality pair. In both cases, the main results of the paper is that the static equilibrium menu remains the unique equilibrium outcome.

4.1. Finite Horizon Price-Quality Differentiation Game

Before we move to the infinite horizon game, as an intermediate step we examine the finite horizon game. Since there is the last period in the finite horizon game, we can apply backward induction. We analyze first what would happen in the last two periods of the game.

Since the low value consumer has zero payoff in the static game, it is straightforward, by backward, induction, to prove that the low value consumer has zero payoff in the two-period game. Sequential rationality, then, dictates that if the monopolist offers price-quality pair which gives him a positive payoff, he surely accepts it. As a consequence, the monopolist can induce the static equilibrium menu in the first period with an arbitrarily small cost. The static equilibrium payoff is, therefore, a lower bound for the equilibrium payoff to the monopolist in two-period game.

Lemma 1. In two-period game, the equilibrium payoff to the monopolist is at least as large as the static equilibrium payoff, $\Pi_s(\mu_0)$.

Proof: Fix an equilibrium. Suppose the monopolist deviates from the equilibrium by offering in the first period the menu, $\{(\theta_1 U(q^*(\theta_1))-(\theta_1-\theta_2)U(q_2(\mu_0))-2\varepsilon, q^*(\theta_1)), (\theta_2 U(q_2(\mu_0))-\varepsilon, q_2(\mu_0))\}$ with $\varepsilon>0$. For $\varepsilon<(\theta_1-\theta_2)(U(q^*(\theta_1))-U(q_2(\mu_0)))$, the incentive constraints holds as a strict inequality for both types of consumer. Furthermore, since the second price-quality pair gives the low value consumer the positive payoff ε , he sure accepts it. Therefore, the high value consumer also surely accepts the first price-quality pair. Otherwise, he reveals his type and his continuation equilibrium payoff should be zero. This deviation gives the monopolist the payoff $\Pi_s(\mu_0)-(1+\mu_0)\varepsilon$. Since ε is arbitrary, the equilibrium payoff to the monopolist should be at least $\Pi_s(\mu_0)$. QED

Lemma 1 shows that the additional opportunity for transaction may potentially benefit the monopolist. However, the static equilibrium payoff is also an upper bound. The monopolist firm cannot outperform the static equilibrium menu at least in two period game.

Lemma 2. In two-period game, for all $(\mu_0, \delta)\in(0,1)\ (0,1)$, the static equilibrium menu is the unique equilibrium outcome.

Proof: We first prove that the game does not last for two period, i.e., the game should end in the first period. Suppose that the game lasts for two periods. Let λ_i be the probability with which θ_i -type consumer rejects the first period menu. In order for the game to last for two period, either $\lambda_1>0$ or $\lambda_2>0$. Suppose that the game reaches the second period with posterior probability $\mu_1=\frac{\lambda_1\mu_0}{\lambda_1\mu_0+\lambda_2(1-\mu_0)}$. Since this is the last period, the monopolist offers the static equilibrium menu with prior probability μ_1, i.e., $\{(\theta_1 U(q^*(\theta_1))-(\theta_1-\theta_2)U(q_2(\mu_1)),q^*(\theta_1)),(\theta_2 U(q_2(\mu_1)),q_2(\mu_1))\}$.

Now consider the first period menu. If the high value consumer waits one period and mimics the low value consumer, he enjoys the surplus, $\delta(\theta_1-\theta_2)U(q_2(\mu_1))$. Hence, by incentive compatibility, the first period price-quality pair for the high value consumer should be $(\theta_1 U(q^*(\theta_1))-\delta(\theta_1-\theta_2)U(q_2(\mu_1)),q^*(\theta_1))$. Let (p,q) be first period price-quality pair for the low value consumer. Then, first it gives zero payoff to the low value consumer, namely, $p=\theta_2 U(q)$. Second, the high value consumer does not have incentive to mimic the low value consumer, $\theta_1 U(q)-\theta_2 U(q)=\delta(\theta_1-\theta_2)U(q_2(\mu_1))$. Namely, $U(q)=\delta U(q_2(\mu_1))$, since θ_1-type consumer accepts the first period price-quality pair with probability (1 - λ_1). The payoff to the monopolist is given as follows:

$$\Pi=\mu_0(1-\lambda_1)[\theta_1 U(q^*(\theta_1))-\delta(\theta_1-\theta_2)U(q_2(\mu_1))-cq^*(\theta_1)]+(1-\mu_0)(1-\lambda_2)(\theta_2 U(q)-cq)$$
$$+\mu_0\lambda_1\delta\{(\theta_1 U(q^*(\theta_1))-(\theta_1-\theta_2)U(q_2(\mu_1))-cq^*(\theta_1)\}+(1-\mu_0)\lambda_2\delta(\theta_2 U(q_2(\mu_1))-cq_2(\mu_2)).$$

By showing that Π is strictly dominated by the static equilibrium payoff, which contradicts Lemma 1, we will prove that the game does not last for two period. For this, we consider several cases separately.

Case 1: $\mu_1\geq\mu^*$

In this case, $q_2(\mu_1)=0$ so that $\Pi=\mu_0(1-\lambda_1)\{\theta_1 U(q^*(\theta_1))-cq^*(\theta_1)\}$ + $\mu_0\lambda_1\delta\{\theta_1 U(q^*(\theta_1))-cq^*(\theta_1)\}$. Note that $\delta<1$, $\theta_1 U(q^*(\theta_1))-cq^*(\theta_1))>0$, and $\lambda_1>0$, otherwise, $\mu_1=0$. $\Pi<\mu_0\{\theta_1 U(q^*(\theta_1))-cq^*(\theta_1)\}$, which cannot exceeds the static equilibrium payoff, $\Pi_s(\mu_0)$. This contradicts Lemma 1.

Case 2: $\mu_1<\mu^*$

In this case, $q_2(\mu_1)>0$. Since $U(q)=\delta U(q_2(\mu_1))$, $U(\cdot)$ is strictly concave and $\delta<1$, $\delta q_2(\mu_1)>q$. Notice the following inequality:

$\theta_1 U(q^*(\theta_1))-\delta(\theta_1-\theta_2)U(q_2(\mu_1))-cq^*(\theta_1)>$ $\theta_1 U(q^*(\theta_1))-(\theta_1-\theta_2)U(q_2(\mu_1))-cq^*(\theta_1)$ >

0.
These inequality implies the following:

$$\Pi < \mu_0\{\theta_1 U(q^*(\theta_1)) - \delta(\theta_1-\theta_2)U(q_2(\mu_1)) - cq^*(\theta_1)\} + (1-\mu_0)(1-\lambda_2)(\theta_2 U(q) - cq)$$
$$+ (1-\mu_0)\lambda_2\delta(\theta_2 U(q_2(\mu_1)) - cq_2(\mu_2))$$
$$= \mu_0\{\theta_1 U(q^*(\theta_1)) - (\theta_1-\theta_2)U(q) - cq^*(\theta_1)\} + (1-\mu_0)(1-\lambda_2)(\theta_2 U(q) - cq)$$
$$+ (1-\mu_0)\lambda_2(\theta_2 U(q) - c\delta q_2(\mu_2))$$
$$\leq \mu_0\{\theta_1 U(q^*(\theta_1)) - (\theta_1-\theta_2)U(q) - cq^*(\theta_1)\} + (1-\mu_0)(\theta_2 U(q) - cq) .$$

The first equality comes from using $U(q)=\delta U(q_2(\mu_1))$ and the second inequality comes from the fact that $\delta q_2(\mu_1) > q$. Note that the last term is nothing but a payoff to the monopolist in static game when it offers the menu, $\{(\theta_1 U(q^*(\theta_1)) - (\theta_1-\theta_2)U(q), q^*(\theta_1)), (\theta_2 U(q), q)\}$. This menu cleary satisfies both participation and incentive constraints, therefore, the payoff cannot exceed the static equilibrium payoff. Hence, $\Pi < \Pi_s(\mu_0)$. This, again, contradicts Lemma 1.

By Case (1) and (2), the game cannot last for two periods. The game should end in the first period. When the monopolist offers a menu in the first period, the menu must
satisfy both participation and incentive constraints. Since the static equilibrium menu gives the highest possible payoff, the menu the monopolist offers in the first period must be the static equilibrium menu which is immediately accepted by both types of consumer. QED.

We now can show, by backward induction, that in finite horizon game, the static equilibrium menu is indeed the unique equilibrium outcome.

Proposition 2. In finite horizon game, for all $(\mu_0, \delta) \in (0,1) \times (0,1)$, the static equilibrium menu is the unique equilibrium outcome.

Proof: To prove uniqueness, we use induction on the number of periods to go. Consider T horizon game. When T=2, Lemma 2 shows that the static equilibrium menu is the unique equilibrium outcome. Suppose that in (T-1) horizon game the unique equilibrium outcome is the static equilibrium menu. In T horizon game with prior belief μ_0 , if the game moves to the next period with posterior μ_1 , by the induction hypothesis, the unique equilibrium outcome in the continuation game is the static equilibrium menu with μ_1 as a prior belief. We now replace the continuation game with its unique equilibrium outcome. The T horizon game, then, reduces to that of two-period game. By Lemma 2, the static equilibrium menu with prior belief μ_0 should be the unique equilibrium outcome in the reduced game, which shows that the static equilibrium menu is indeed the unique equilibrium outcome in the whole T horizon game. QED.

4.2. Infinite Horizon Price-Quality Differentiation Game

We now analyze the infinite horizon game. Although we cannot apply backward induction in infinite horizon game, it still holds that the equilibrium payoff to the low value consumer is zero.

Lemma 3. Let $u^* =$ sup $\{\theta_2 U(q) - p\}$ where supremum is taken over all equilibrium outcomes for the low value consumer after all histories and all prior probabilities. Then, $u^* = 0$.

Proof: By the participation constraint, $u^* \geq 0$. Suppose $u^* > 0$. Then, there exists a history h_t with posterior belief μ which is reached with positive probability. Suppose that in equilibrium, conditional on h_t , the monopolist offers a menu M which gives the low value consumer u with $0 < \delta u^* < u$. By rejecting M, the low value consumer can attain at best δu^* . Since $u > \delta u^*$, the low value consumer accepts M. The high value consumer also accepts M otherwise he reveals his type and receives zero payoff in the continuation equilibrium. Therefore, conditional on h_t , when the monopolist offers M, this menu is accepted by both types of consumers and the game ends.

Let (p_2, q_2) be the price-quality pair in M which gives the low value consumer the payoff u , i.e., $\theta_2 U(q_2) - p_2 = u > \delta u^*$. Suppose that conditional on h_t , instead of offering M, the monopolist deviates in the following way. The monopolists offers $(p_2 + \varepsilon, q_2)$ intended for the low value consumer where $\varepsilon > 0$ is chosen such that $\theta_2 U(q_2) - p_2 - \varepsilon > \delta u^*$. To the high value consumer, the monopolist offers $(\theta_1 U(q^*(\theta_1)) - \theta_1 U(q_2) + p_2 + \varepsilon, q^*(\theta_1))$. For all $\varepsilon > 0$, the high value consumer strictly prefers $(\theta_1 U(q^*(\theta_1)) - \theta_1 U(q_2) + p_2 + \varepsilon, q^*(\theta_1))$ to $(p_2 + \varepsilon, q_2)$, and for sufficiently small $\varepsilon > 0$, the low value consumer strictly prefers the latter to the former. Since $(p_2 + \varepsilon, q_2)$ gives the low value consumer the payoff higher than δu^* , he surely accepts it. Also does the high value consumer accept $(\theta_1 U(q^*(\theta_1)) - \theta_1 U(q_2) + p_2 + \varepsilon, q^*(\theta_1))$. We now show that this deviation gives the monopolist higher payoff than M does. If the consumer is of low value, since the monopolist sells the product of the same quality at a higher price, it clearly increases the payoff. Suppose the consumer is

of high value. Let $(p_1, q^*(\theta_1))$ be the price-quality pair in M chosen by the high value consumer. Then,
by incentive compatibility, $\theta_1 U(q^*(\theta_1)) - p_1 \geq \theta_1 U(q_2) - p_2$. This implies that $p_1 \leq \theta_1 U(q^*(\theta_1)) - \theta_1 U(q_2) + p_2 < \theta_1 U(q^*(\theta_1)) - \theta_1 U(q_2) + p_2 + \varepsilon$. Hence $(\theta_1 U(q^*(\theta_1)) - \theta_1 U(q_2) + p_2 + \varepsilon, q^*(\theta_1))$ gives the higher payoff to the monopolist than $(p_1, q^*(\theta_1))$ does. With the high value consumer, the payoff to the monopolist also increases. Therefore, this deviation gives the higher payoff to the monopolist than the equilibrium does. This is a contradiction, thereby, μ^* should be zero. QED.

Since the low value consumer accepts any price-quality pair which gives him a strictly positive payoff, by the same way in proving Lemma 1, the static equilibrium payoff is still a lower bound for the equilibrium payoff to the monopolist in the infinite horizon game.

Lemma 4. In infinite horizon game, the equilibrium payoff to the monopolist is at least
as large as the static equilibrium payoff, $\Pi_0(\mu_0)$.

Our main result is that the monopolist cannot outperform the static equilibrium menu in infinite horizon game, either. Next lemma shows that in equilibrium, the game ends with a bounded number of periods.

Lemma 5. In any equilibrium, with probability 1, the game ends within a bounded number of periods.

Proof: Fix an equilibrium. We show that there exists T < ∞ such that the game ends within T periods. Let $T_e =$ sup{ $t \mid (p, q, t)$ is an equilibrium outcome for the high value consumer.}. First we show that $T_e < \infty$. Suppose not. Then, in equilibrium, in every period the high value consumer rejects the menu with positive probability. Let T^* = inf{ $T \mid \forall t \geq T$, the high value consumer rejects the menu with probability 1}. If T^* = ∞, then in equilibrium, the high value consumer accepts the menu with positive probability infinitely many times. Hence for all n, there exits $t_n > n$ such that, in equilibrium, with positive probability thei game ends in (p_n, q_n, t_n) for the high value consumer. Then, the high value consumer should be indifferent among these outcomes. Hence, the equilibrium payoff to the high value consumer is

$$\delta^{t_1-1}(\theta_1 U(q_{t_1}) - p_{t_1}) = \delta^{t_2-1}(\theta_1 U(q_{t_2}) - p_{t_2}) = \cdots = \delta^{t_n-1}(\theta_1 U(q_{t_n}) - p_{t_n}) = \cdots .$$

Since the maximum surplus is finite and δ^{t_n} goes to zero as n goes to ∞, the equilibrium payoff to the high value consumer should be zero. In equilibrium, by incentive compatibility, the low value consumer either rejects all menus or choose the price-quality pair, (0,0). Therefore, the equilibrium payoff to the monopolist is strictly less than $\mu_H\{\theta_1 U(q^*(\theta_1)) - cq^*(\theta_1)\}$, which, in turn, does not exceed the static equilibrium payoff, $\Pi_s(\mu_0)$. This contradicts Lemma 4.

If T^* < ∞, then the high value consumer eventually rejects all menu. The equilibrium payoff to him, therefore, should be zero. Again, by incentive compatibility, the low value consumer either rejects all menus or choose the price-quality pair, (0,0).
Therefore, the equilibrium payoff to the monopolist is strictly less than $\mu_H\{\theta_1 U(q^*(\theta_1)) - cq^*(\theta_1)\}$. This, again, contradicts Lemma 4. Hence $T_L < \infty$. Now, if the game last more than T_L periods, the monopolist knows that the consumer is of low value. The game then must end in the next period because the monopolist offers $(\theta_2 U(q^*(\theta_2)), q^*(\theta_2))$ which is immediately accepted by the low value consumer. The game ends, therefore, within T_L +1 periods. QED.

We now state and prove the main result for the infinite horizon game.

Proposition 3. In infinite horizon game, for all $(\mu_0, \delta) \in (0,1)\ (0,1)$, the static equilibrium menu is the unique equilibrium outcome.

Proof: Fix an equilibrium. Let T = max{ $t \mid (p, q, t)$ is an equilibrium outcome.} By Lemma 7, T < ∞. We will show that T = 1. Suppose T > 1. Then, in equilibrium, with positive probability the game reaches a history of length T-1, $h_{T-1} = (h_{T-2}, M_{T-2})$ with posterior probability μ_{T-1}. Suppose that conditional on h_{T-1}, the monopolist offers the menu, M_T = { $(p_1^T, q_1^T), (p_2^T, q_2^T)$ }. Since the game ends by period T, M_T must be accepted by both types of consumer. Note that conditional on h_{T-1}, the continuation equilibrium payoff to the monopolist is bounded below by the static equilibrium payoff, $\Pi_s(\mu_{T-1})$. Since this payoff is attained in a way that satisfies both participation and incentive constraints, M_T should be the static equilibrium menu with prior probability μ_{T-1}. In period T-1 with history h_{T-2} and the posterior probability μ_{T-2}, in equilibrium, the monopolist offers M_{T-1} which is rejected with positive probability and the game moves to the next period with posterior belief μ_{T-1}. Then, the game will end with the static equilibrium menu with prior probability μ_{T-1}. By Lemma 2, however, it will pay the monopolist to finish the
game in period T-1 by offering the static equilibrium menu with prior probability μ_{T-2}. This contradicts the fact that the game last for T periods. Hence T must

be equal to 1. Suppose that the monopolist offers a menu, $\{(p_1, q_1), (p_2, q_2)\}$ in the first period. Then, this menu is immediately accepted by both types of consumer. The monopolist's equilibrium payoff is bounded below by the static equilibrium payoff. Since this payoff must be achieved in a way that satisfies both participation and incentive constraint, and since the static equilibrium menu is unique, $\{(p_1, q_1), (p_2, q_2)\}$ must be equal to the static equilibrium menu and this menu gives the monopolist the static equilibrium payoff, $\Pi_s(\mu_0)$. QED.

5. Conclusion

This paper examines the dynamic incentive of price-quality differentiation by a monopolist firm. The price-quality differentiation by a monopolist firm is called the second degree price discrimination or non-linear pricing. In the literature, the problem of non-linear pricing is mostly analyzed in static model based upon the assumption that the monopolist firm can offer on a take-it-or-leave-it basis. In reality, in particular, for the durable goods, it is dubious whether the monopolist firm has indeed such a commitment power. This paper extends the problem of non-linear pricing in the dynamic perspectives. Once the menu is rejected, the monopolist can offer another menu in the next period. The main result is that the equilibrium in dynamic price-quality differentiation game is exactly the same as the static equilibrium. Therefore, our result justifies the static analysis.

The model considered here is somewhat restrictive; it considers two types only, one-sided uncertainty exists only, and the informed consumer cannot make a counteroffer. The extensions to one of each direction, continuum type, two-sided uncertainty and alternating offers seem challenging research agenda. The models with these features await further investigation.

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